4/26/18 "Too many of us are not living our dreams because we are living our fears."-Les Brown

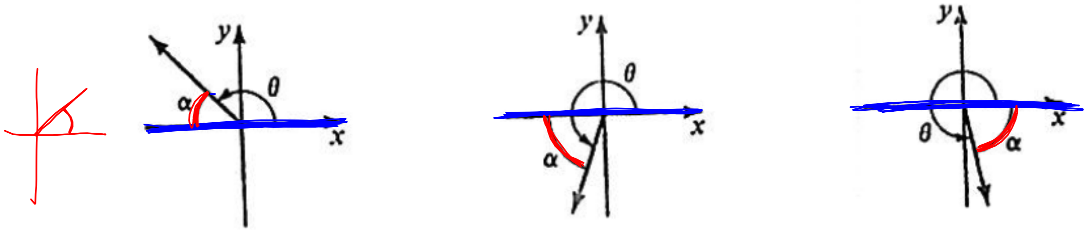
HW: "Angles" Exercise Set B, C, D even numbers only
Test 1 on Wednesday 5/2

AIM: What are Reference Angles?

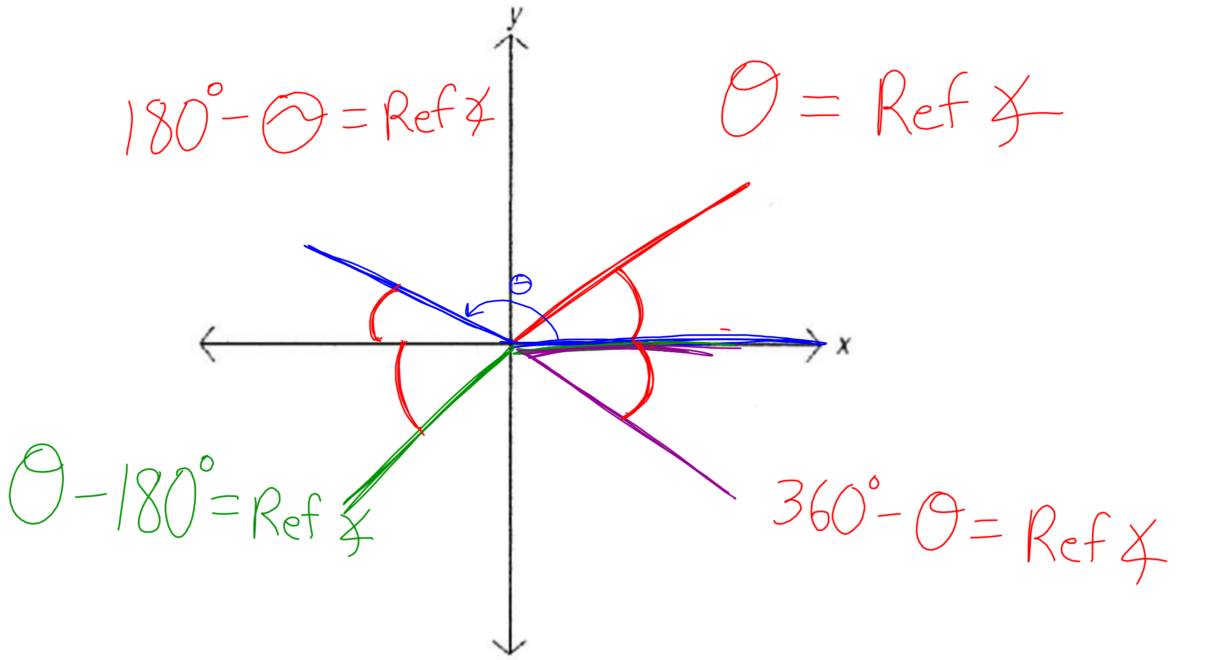
Warm Up:

✱ Positive acute ∡ between 0° and 90°

Given an angle θ in standard position, the ***reference angle* of θ**, is the positive acute angle formed by the terminal side of θ and the positive or negative portion of the x-axis. In each of the following figures, angle α is the reference angle of angle θ.

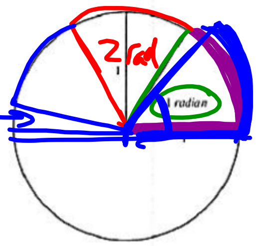


Reference angles will help you to express the sine, cosine or tangent of any angle in terms of the sine, cosine or tangent of a positive acute angle. ✱ $0 \le \theta \le 360$



The degree is not the only unit for measuring angles. Another unit, called the radian, is more commonly used in advanced mathematics. An angle of 1 ***radian*** (drawn in standard position, measured counterclockwise) is a central angle that intercepts an arc equal in length to the radius of the circle.

$180^\circ = \pi$ radians .14



An angle of 2 radians intercepts an arc twice as long as the radius of the circle, and so on. In general, an angle θ (theta) radians will intercept an arc θ times as long as the radius.

If θ is the measure in radians of a central angle of a circle, s is the length of the intercepted arc, and r is the radius of the circle, then:

$$s = |\theta| r \quad \text{or} \quad |\theta| = \frac{s}{r}$$

~~Let's find a conversion from radians to degrees:~~

~~Since a circle of radius 1 has a circumference of __________.~~

~~Substituting into the formula~~ $\cancel{s = r\theta}$

~~Therefore: =~~

=

=

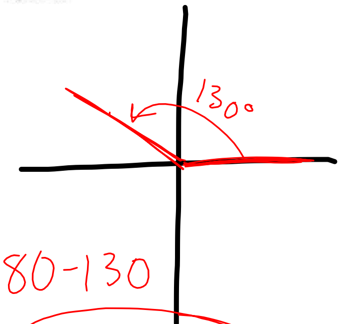
* To convert from radians to degrees. Multiply by $\frac{180}{\pi}$

* To convert from degrees to radians Multiply by $\frac{\pi}{180}$

a. Sketch the given angle in standard position, then sketch its reference angle. Label the reference angle as angle α.

b. State the measure of angle α.

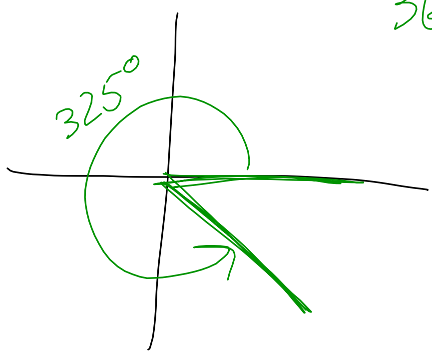
1. 130°



$180-130$

$50° = \alpha$

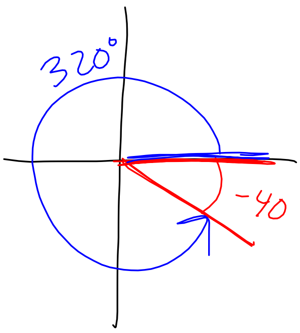
3. 325°



$360-325=$

$35° = \alpha$

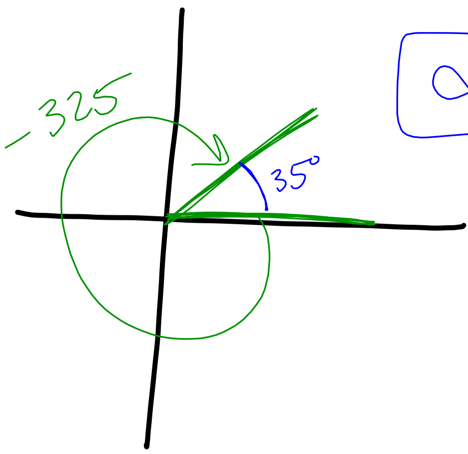
5. −40° $+360° = 320°$



$360-320$

$\boxed{\alpha = 40°}$

7. −325° $+360 = 35°$

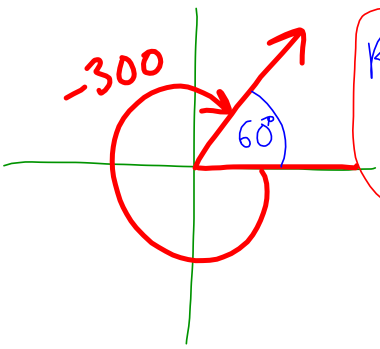


$\boxed{\alpha = 35°}$

9. $-\frac{5\pi}{3} \cdot \frac{180}{\pi}$

$\frac{-5(\overset{60}{\cancel{180}})}{\cancel{3}} = -300 + 360$

$= 60$



Ref ∡ = 60°

Ref ∡ = $\frac{\pi}{3}$

Convert to radians

$\cancel{60} \cdot \frac{\pi}{\underset{3}{\cancel{180}}} = \boxed{\frac{\pi}{3}}$

In 1–9, express in degrees the angle whose radian measure is given.

1. $\frac{4\pi}{3} \cdot \frac{180}{\pi} = \boxed{240^\circ}$

3. $\frac{2\pi}{5} = \frac{2(180)}{5} = \boxed{72^\circ}$

5. $\frac{5\pi}{9}$ $\frac{5(180)}{9} = \boxed{100^\circ}$

In 11–19, express in radians the angle whose degree measure is given.

11. 15° $\cdot \frac{\pi}{180} = \frac{15\pi}{180} = \frac{1\pi}{12}$

OR

$\frac{\pi}{12}$

13. 105° $\cdot \frac{\pi}{180} = \frac{105\pi}{180}$

$\frac{7\pi}{12}$

15. 160° $\cdot \frac{\pi}{180}$

$\frac{160\pi}{180} = \frac{8\pi}{9}$

$\frac{8}{9}\pi$

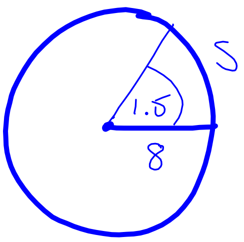
$s = \theta \cdot r$

$\theta = \angle$ in radians

r = radius

s = arc length

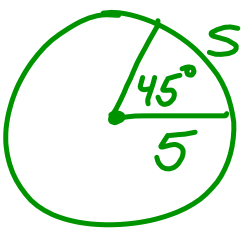
1. In a circle of radius 8 inches, find the length of the arc intercepted by a central angle of 1.5 radians.



$s = (1.5)(8)$

$\boxed{s = 12 \text{ inches}}$

3. In a circle with radius 5 inches, find the length of an arc intercepted by a central angle of 45°.



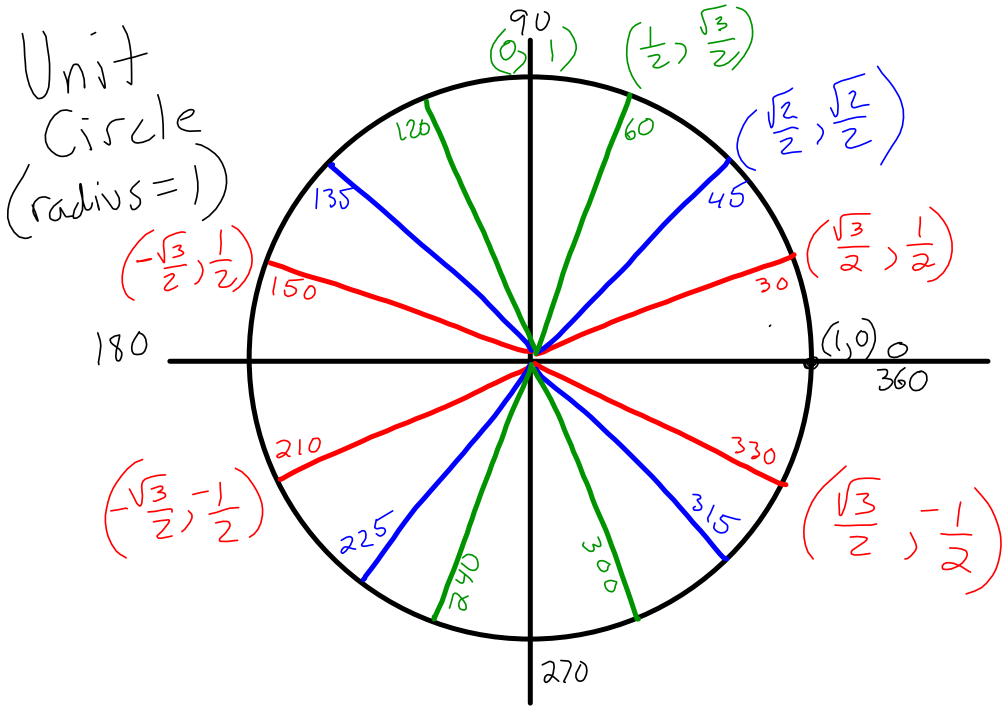
Convert 45° to radian

$45 \cdot \frac{\pi}{180} = \frac{1\pi}{4}$

$s = \theta r$

$s = \left(\frac{\pi}{4}\right)(5)$

$s = \boxed{\frac{5\pi}{4} \text{ inches}}$


Unit
Circle
(radius = 1)
90
(0, 1)
$(\frac{1}{2}, \frac{\sqrt{3}}{2})$
$(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
$(\frac{\sqrt{3}}{2}, \frac{1}{2})$
$(-\frac{\sqrt{3}}{2}, \frac{1}{2})$
120
60
135
45
150
30
180
(1, 0)
0
360
210
330
225
315
240
300
$(-\frac{\sqrt{3}}{2}, -\frac{1}{2})$
$(\frac{\sqrt{3}}{2}, -\frac{1}{2})$
270